On the "Non-Restricted special Relativity" theory (NRR) and further comments on "Cherenkov vs X-waves"

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Abstract — Our aim in this paper is to recall some essential points of "Extended special Relativity", now more correctly called "Non-Restricted special Relativity" theory (NRR), and in particular of the Extended Maxwell Equations; as well as to set forth some further comments on the *basic* differences between Cherenkov Radiation and the so-called X-shaped Waves, met within the more recent realm of the Non-diffracting Waves (also known as Localized Waves). The occasion is furnished by some very recent Seshadri's comments[1] on a previous article of ours, titled "Cherenkov radiation versus X-shaped localized waves" (see[2], and arXiv:0807.4301[physics.optics]), and not less on NRR itself.

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1. General observations

Our aim in this paper is to recall some essential points of "Extended special Relativity", now more correctly called "Non-Restricted special Relativity" theory (NRR), and in particular of the Extended Maxwell Equations; as well as to set forth some further comments on the *basic* differences between Cherenkov Radiation and the so-called X-shaped Waves, met within the more recent realm of the Non-diffracting Waves (also known as Localized Waves). The occasion is furnished by some very recent Seshadri's comments[1] on a previous article of ours, titled "Cherenkov radiation versus X-shaped localized waves" (see[2], and arXiv:0807.4301[physics.optics]), and not less on NRR itself.

Let us first answer Seshadri's comments[1] to the previous article of ours, titled "Cherenkov radiation versus X-shaped localized waves"[2].

Actually, as we were saying, Seshadri's comments do not refer too much to our article[2], whose aim was mainly showing in detail that X-shaped localized waves have nothing

to do with Cherenkov radiation, as maintained by contrast in Ref.[3]. It should be pointed out that the so-called Localized Waves (LW) are nondiffracting ("soliton-like") solutions to linear wave equations, and a host of theoretical, mathematical, numerical-simulation, and experimental works have demonstrated them to exist with subluminal, luminal or superluminal peak-velocities V (see, e.g., [4] and references therein). These wave packets are remarkable and have been thoroughly investigated more for their limited-diffraction and self-reconstruction properties than for their group-velocities. However, merely for mathematical and experimental[5,6] reasons, the ones that drew more attention were the superluminal "X-shaped" ones, which in fact started to be considered even for practical applications in 1992. The X-waves have the shape of a double cone. For this reason, some authors have been tempted —let us confine ourselves to electromagnetism— to look for links between them and the Cherenkov radiation: a link that we have throroughly shown in [2] to be untenable.

The comments in Ref.[1] are *partially* related with our article [2]; but in reality they appear to be more addressed to a previous paper of ours, with the rather different title (and subject) "Localized X-shaped field generated by a superluminal charge"[7]: A very different subject, indeed, since even the superluminal LWs are known[4] to consist of soperpositions of "lateral" (feeding) waves, which travel at the ordinary speed of light[8]. Actually, in our article [2] we were induced to go briefly back to questions faced by us in Ref.[7], and to restate one of those questions more rigorously in terms of half-retarded and half-advanced Green functions, *only* because a part of Ref.[3] itself was inadvertently pointing in that direction.

The criticisms in Ref.[1] regard moreover the *theory of Extended Special Relativity, correctly called by Seshadri Non-Restricted Special Relativity* (NRR), which, on the basis of the ordinary postulates of Special Relativity (SR), chosen of course in a modern way, had shown how the theory of SR in its Einstein-Minkowski formulation could have pre-

dicted already in 1908 the existence of antiparticles, on one hand, as well as described superluminal motions without severe violations or paradoxes, on the other hand. Incidentally, superluminal motions were investigated in pre-relativitic times by J.J.Thomson (1889), O.Heaviside (1892), A.Des Coudres (1900), and particularly, in 1904 and 1905, by Sommerfeld[9,10]. A brief resume of the very stimulating results about a radiating superluminal charge determined, e.g., by Sommerfeld can be found in [11]. In post-relativistic times, the construction of a NRR started with Sudarshan *et al.*'s papers[12,13], and was continued by one of the present authors and coworkers (see, e.g., review [14] and refserences therein). For future use, let us summarize some characteristics[14] of NRR, an interesting theory that allows a deeper understanding of physics (cf., e.g., Refs.[15,16]),as we shall see, even if tachyons would not exist as asymptotically free objects in our cosmos. A modern choice of the postulates of SR is known to be: (i) Principle of relativity; (ii) space-time homogeneity and space isotropy. Form these two postulates it has been demonstrated since 1911 that the existence of one, and only one, *invariant* speed follows; and experience tells us it to be the speed c of light in the vacuum. As a consequence, following, e.g., Landau, one gets

$$ds'^2 \;=\; \pm ds^2 \tag{1}$$

ds^2 being the square of the spacetime distance element, that is, the four-dimensional quantity $ds^2 \equiv dx_\mu dx^\mu \equiv dt^2 - d\boldsymbol{x}^2$, where $d\boldsymbol{x}^2 \equiv dx^2 + dy^2 + dz^2$. By choosing the sign plus (+), one describes the subluminal world, while the superluminal one is described[14] when choosing the sign minus (-). Let us notice, therefore, that also in NRR the ds^2 is invariant, except for a sign. More generally, all the quadratic forms are analogously invariant: for instance, in the simplest cases, $x'_\mu x'^\mu \;=\; \pm x_\mu x^\mu$; $p'_\mu p'^\mu \;=\; \pm p_\mu p^\mu$; $x'_\mu p'^\mu \;=\; \pm x_\mu p^\mu$; the minus sign entering into play when passing on from a subluminal object (bradyon, B) to a superluminal object (tachyon, T). It is on the basis of Eqs.(1), and of some

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obvious further assumptions, that the ordinary subluminal Lorentz transformations (LT) are written down in correspondence with the sign +, and analogously the superluminal Lorentz "transformations" (SLT) in correspondence with the sign -. We shall come back again to this point.

Let us forget for a moment about tachyons, and consider only the sign + in Eqs.(1), with a subluminal boost along x. Since conditions (1) are quadratic, each Lorentz transformation L can be taken with a double sign [this is another double sign yielded by mathematics: which has nothing to do with the one in Eqs.(1)]. A little more formally, let us recall that the set of all subluminal LTs consists of four pieces, which form a non-compact, non-connected group (the Full Lorentz Group). Wishing to confine ourselves to spacetime "rotations" only, i.e., to the case det $L = +1$, we are left with two pieces only: $\{L_+^\uparrow\}$, and $\{L_+^\downarrow\}$, which give origin to the group of the proper (orthochronous *and* antichronous) transformations $\mathcal{L}_+ \equiv \mathcal{L}_+^\uparrow \cup \mathcal{L}_+^\downarrow \equiv \{L_+^\uparrow\} \cup \{L_+^\downarrow\}$, and to the subgroup of the (ordinary) proper orthochronous transformations $\mathcal{L}_+^\uparrow \equiv \{L_+^\uparrow\}$. In other words, $\mathcal{L}_+$ can be written as

$$\mathcal{L}_+ = \mathcal{L}_+^\uparrow \otimes Z(2) \quad ; \quad Z(2) \equiv \{\sqrt{+1}\} \equiv \{+1, -1\} . \tag{2}$$

Let us skip in the following, for simplicity's sake, the subscript +. Given an orthochronous transformation $\overline{L}^\uparrow \in \mathcal{L}_+^\uparrow$, another antichronous transformation $\overline{L}^\downarrow$ always exists such that $\overline{L}^\downarrow = (-1)\overline{L}^\uparrow$, for all $\overline{L}^\downarrow \in \mathcal{L}_+^\downarrow$, and vice-versa. Such a one-to-one correspondence does obviously allow one to write $\mathcal{L}_+^\downarrow = -\mathcal{L}_+^\uparrow$. Usually, even the latter piece is discarded, since the antichronous LTs change the time direction sign (something that is not acceptable). But any LT acts also on the dual four-dimensional space, that is, on the energy-momentum space, changing the energy sign too (something not acceptable as well). *But* it has been shown long time ago, in a large number of papers (see, e.g., Refs.[12-14,17] and refs.

therein), that those two paradoxical occurrences, when they are —as they actually are— simultaneous, lead necessarily to the conclusion that the antichronous LTs describe the corresponding antiparticles (travelling of course forward in time, with positive energy), an orthodox conclusion, that could have been inferred in 1908, and that has been confirmed by the experimental discovery of antiparticles. Namely, the theory of SR, once based on the whole proper Lorentz group and not only on its orthochronous part, describes a Minkowski spacetime populated by both matter and antimatter.

What is more important for us is that, *going now back to the superluminal case* [minus sign in Eqs.(1)], also the SLTs are to be taken with a double sign. The inverse transformations do once more exist, contrary to a claim in Ref.[1]. The situation in this case is however more complex, since, e.g., two successive SLTs yield an ordinary LT; actually, the group $\mathcal{G}$ of all (subluminal or superluminal) transformations can be formally written, with $Z(4) \equiv \{\sqrt[4]{+1}\}$, as

$$\mathcal{G} = \mathcal{L}_+^{\uparrow} \otimes Z(4) . \tag{3}$$

We shall come back again to this point too. In any case, the Stueckelberg-Feynman-Sudarshan (SFSR) switching principle, invoked before, does work also in the case of SLTs, even if in this case the distinction particle/antiparticle is no longer Lorentz invariant[12-14]. See the two-sheeted hyperboloids (case of Bs), and the one sheeted hyperboloid (case of Ts), depicted for instance in figure 5 of Ref.[14], and here reproduced as our Fig.1. In this figure, it is the symmetry with respect to the plane $E = 0$ which leads from particles to antiparticles (namely, from Bs to anti-Bs in the two-sheeted case, and from Ts to anti-Ts in the one-sheeted case). Let us add that the SFSR reinterpretation procedure not only can, but *must* be applied[12-14,17].

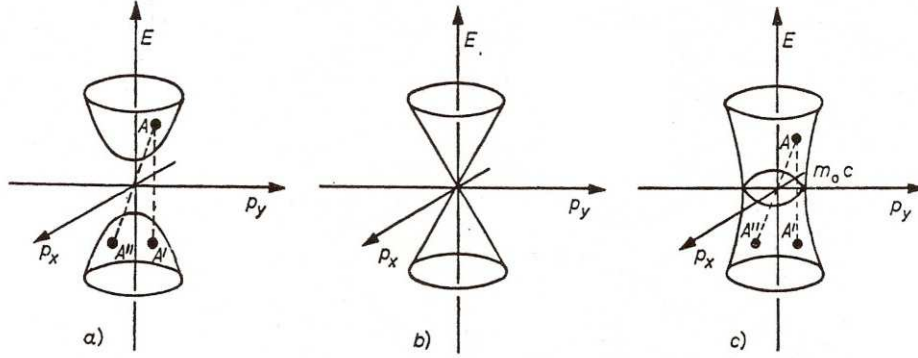Below, we shall need another result of NRR. By implementing the aforementioned rule

Figure 1: Pictures (in 3D only) of the surfaces $p^2 \equiv E^2 - \boldsymbol{p}^2 = \pm m_0^2$: In figure a) for bradyons, when $p^2 > 0$; in b) for luxons, when $p^2 = 0$; and in c) for tachyons, when $p^2 < 0$. The symmetry matter/antimatter[14] is the symmetry w.r.t. the plane $E = 0$: See the text.

that SLTs change the sign of all the quadratic forms, in Refs.[18,14] it was demonstrated that, if an object is a sphere (or just a point) when at rest, it will appear, when traveling with superluminal speed V, as the region contained between an indefinite double-cone (with semi-angle α given by the simple relation $\tan \alpha = \sqrt{V^2 - 1}$) and an internal two-sheeted hyperboloid (or just as a double cone): See Fig.2, reproduced from Refs.[18,14]. Notice that this holds, even if the speed c of light is a limiting speed, which in both SR and NRR cannot be crossed, neither from the left nor from the right[14].

2. Specific replies

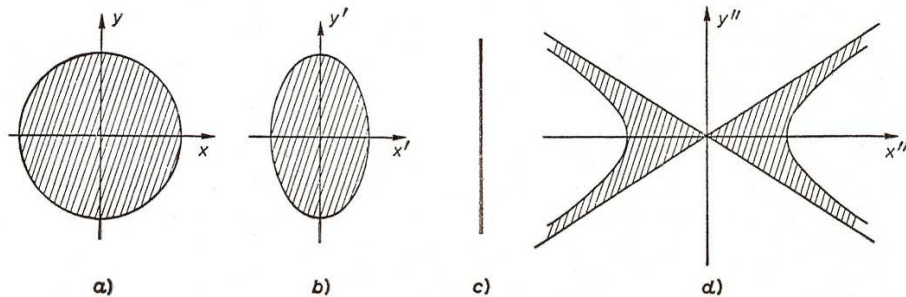### 2.1 Reply to Seshadri's Section 2

Figure 2: An intrinsically spherical (or pointlike, at the limit) object appears in the vacuum as an ellipsoid contracted along the motion direction when endowed with a speed $v < c$. By contrast, if endowed with a speed $V > c$ (even if the c-speed barrier cannot be crossed, neither from the left nor from the right), it would appear[14,18] no longer as a particle, but as occupying the region delimited by a double cone and a two-sheeted hyperboloid —or as a double cone, at the limit–, and moving with Superluminal speed V [the cotangent square of the cone semi-angle, with $c = 1$, being $V^2 - 1$. For simplicity, a space axis is skipped. This figure is taken from our Refs.[14,18].

The Wheeler-Feynman approach –just optional in the case of photons– becomes necessary in the case of tachyons, so that one has to consider half advanced and half retarded potentials, as we did, e.g., in Subsection 2D of Ref.[2] (and in [7]). Then, one gets that a superluminal charge does not emit Cherenkov radiation in the vacuum (as it must obviously be the case in classical physics, since that radiation is induced by the charge from a medium).

In performing the integration in the complex ω plane, with $\zeta \equiv z - vt$, Seshadri[1] goes on to $\omega_r + i\,\omega_i$, so that his integration is essentially along the real axis, but indented from above at the two existing poles whenever the imaginary part ω_i is assumed to be positive; afterwards, by closing the integration contour in the upper plane, he gets of course that for $\zeta > 0$ the integral yields zero. *However,* if one chooses a negative ω_i, the integration in the upper part of the complex plane ($\zeta > 0$) does not yield zero! The choice depends on the imposed physical conditions, ours being that the superluminal

charge should not radiate when traveling with constant speed in the vacuum; for physical reasons already mentioned in 1973 in old papers like [19] [but indeed discussed among the "superluminal" scholars (like E.C.G. Sudarshan, and Recami) since 1967]. Cf. figure 2 of Ref.[2], here reproduced as Fig.3, depicting how the total energy flux crossing a large spatial surface containing the charge itself is just zero. Our aforementioned choice has been implemented[2,7] by adding half retarded and half advanced Green function: which implies the existence of both the rear and the front cones. [Incidentally, Seshadri's choice (unacceptable for physical reasons) refers in a sense to a limiting case of the initial value problem discussed in Ref.[20].]
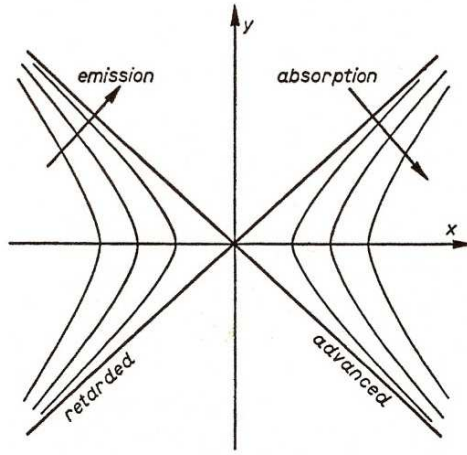

Figure 3: The spherical equipotential surfaces of the electrostatic field created by a charge at rest get transformed into two-sheeted rotation-hyperboloids, contained inside an unlimited double-cone, when the charge travels at superluminal speed[14]. This figures shows, among the others, that a Superluminal charge traveling at constant speed, in a homogeneous medium like the vacuum, does *not* lose energy. See the text.

Let us mention, incidentally, that interesting results have been found even in the case of a supersonic point source traveling in a fluid at rest, with respect to a static observer[21,22].

Let us add that one must have recourse to the Wheeler-Feynman approach, since,

when in the presence of a tachyon, *laws* such as Retarded Causality are required to be covariant by the Principle of Relativity, but *description details* as the emitter/absorber labels do not have to be (and are not) invariant[14,17]. This is a known consequence of the very SFSR reinterpretation procedure, which enters the play when solving[14,17,23] the so-called causal paradoxes (for a thorough solution of the causal paradoxes associated with tachyonic motions, see Ref.[17]).

Before going on, let us stress that the Non-restricted Theory of Special Relativity allows describing also antimatter and superluminal motions: But this, as already mentioned, does not mean *a priori* that tachyons must exist as asymptotically free objects in our cosmos; for reviews of the actual experimental possibilities, one could look, e.g., at Refs.[24,25].

2.2 Reply to Seshadri's Section 3

This is a correct criticism, in the sense that, because of a misprint, we wrote down a wrong sign for the result of the contested integral. But our minus (instead of plus) sign does not influence outgoing or incoming waves since the physically meaningful quantities are obtained through the Poynting vector, which does not depend on that sign.

2.3 Reply to Seshadri's Section 4

As mentioned above, the general Lorentz transformations (GLT) form a group, and the inverse transformations exist always[14]. The SLTs form a group not by themselves,

but together with the ordinary LTs: Let us repeat that, in fact, two successive SLTs constitute an ordinary LT, for physical reasons easily understandable on the basis of NRR, and in particular of its "duality principle" (cf., for example, Sec.5 in Ref.[14], which is downloadable for instance from the site www.unibg.it/recami).

The transformations (6) and (7) of Ref.[1], invented by its author, are not acceptable as SLTs: actually, it can be easily verified that they imply $ds'^2 = +ds^2$, which refers to the case of the ordinary (subluminal) LTs. Indeed, the fundamental relation (1), as we said, reads

$$ds'^2 \;=\; \begin{cases} +ds^2 & \text{for } v^2 < c^2 \text{ ;} \\ -ds^2 & \text{for } v^2 > c^2 \text{ .} \end{cases}$$

All the following considerations in [1], therefore, are potentially incorrect. In particular, as we have already seen, the SLTs do admit an inverse transformation[14]. We are grateful, nevertheless, to Seshadri for the attention he kindly paid to our SLTs[14], and to our extension of Maxwell equations. Indeed, the calculations in Sec.4 of Ref.[1], relative to the Maxwell equations, are in the right direction, and reproduce the generalized Maxwell equations, written by Mignani and Recami at the beginning of the seventies, and rewritten, e.g., as Eqs.(205) in the 1986 Ref.[14].

Since these are rather interesting results, let us seize the present opportunity for re-calling our point with respect to the Maxwell equations. Let us first of all observe that the electric charge ought to be actually called "electromagnetic charge" since it creates also a magnetic field as soon as it moves. Then, it loses meaning to look for magnetic charges, that is, for ordinary magnetic poles, which would be inconsistent with the Universality of the electromagnetic interactions. The Maxwell equations would however remain (inel-egantly) incomplete and non-symmetric. The solution in principle is the following: If one insists in calling electric the subluminal charge —which, in suitable units, creates more electric that magnetic field components—, then you can call magnetic the superluminal charge —which creates more magnetic than electric field components (see figure 46 of

Ref.[14], here reproduced as our Fig.4). Indeed, the superluminal charges happen to contribute to Maxwell equations just in the places where a contribution was expected from magnetic monopoles.
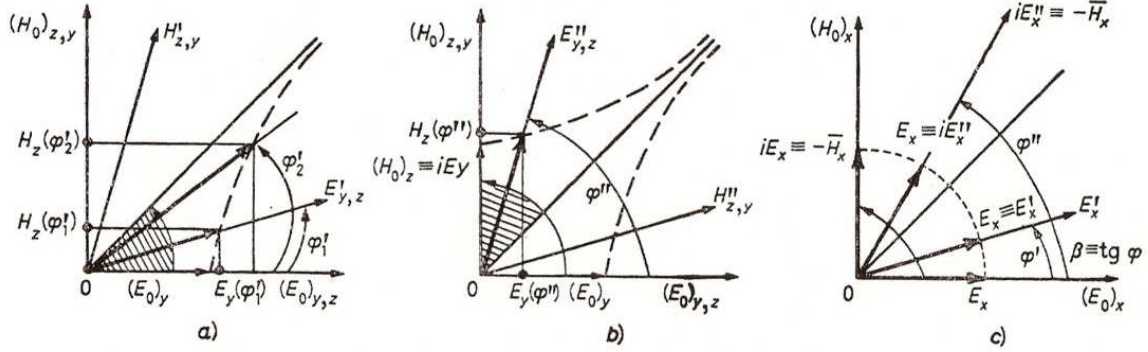

Figure 4: Let us consider in a sistem af rest a purely electric uniform field $\boldsymbol{E}$ parallel, e.g., to y. When moving along x with respect to that system, one observes also a magnetic field along z: Figure a) depicts the ordinary subluminal case with $v \equiv v_x < c$, in which case (in Heaviside-Lorentz units) it is $H_z < E_y$. When moving superluminally with $V \equiv V_x > c$, the magnetic field H_z becomes[14] (in the same units) larger than the electric field E_y: See Figure b); and when $V \to \infty$ we are left with a purely magnetic field[14]. See the text.

Going back to the form of the SLTs, let us add that the fundamental theorem of NRR is [with $u^2 < 1$; $U^2 \equiv 1/u^2 > 1$; and $c = 1$]:

$$\mathrm{SLT}(\boldsymbol{U}) = \pm \mathcal{S} \cdot \mathrm{LT}(\boldsymbol{u}) \,, \tag{4}$$

where the velocities $\boldsymbol{U}$ and $\boldsymbol{u}$ are parallel, and[14] $\mathcal{S} \equiv \mathcal{S}_4 = i\mathbb{1}$. In 2D the theory becomes very simple, rather interesting and rich of useful consequences[14,26] (at the extent that somebody believed it to be so elegant, as to be necessarily true...). In 4D, however, the best mathematical expression for the SLTs is an open problem (see subsections 14.1–14.7 of Ref.[14]). In fact, our Fig.2 shows that SLTs should transform points into double cones, that is, manifolds into manifolds. Anyway, Barut and Chandola[27], reasoning in terms

of mere analytical continuations, adopted for the SLTs in 4D a (very formal) expression of the type $[c = 1; \ U^2 > 1; \ \gamma \equiv 1/\sqrt{U^2 - 1}]$:

$$\pm \, \boldsymbol{x}' \ = \ i\boldsymbol{x} + \frac{\gamma - 1}{U^2}\boldsymbol{U}(\boldsymbol{U} \cdot \boldsymbol{x}) - \gamma\boldsymbol{U}t \ ; \qquad \pm t' \ = \ \gamma(t - \boldsymbol{U} \cdot \boldsymbol{x}) \ , \tag{5}$$

without meeting any difficulties. We shall come back to this problem in our next Subsection, when replying to Seshadri's Sec.5.

2.4 Reply to Seshadri's Section 5

Let us immediately say that Eqs.24 and 25 in Ref.[1] are not at all the "original superluminal Lorentz transformations" characterizing NRR, as claimed by the author, who does not seem to be aware of Ref.[14]. These equations are patently not Lorentz covariant, because they do not satisfy Eq.(1). (Only in the 2D case they result to be not far from the correct ones, which are already known[14,26]). Incidentally, such unsatisfactory relations, supposed to be SLTs, were proposed several times in a number of papers that appeared in the seventies and eighties, and were regularly confuted.

It may be somewhat odd to see naive, wrong equations like (24, 25) of Ref.[1] associated with NRR, a theory that, even if not yet perfect from the *formal* point of view, has been (thoughtfully) developed along decades, from 1962 (Ref.[12]) to at least 1986 (Ref.[14]).

Let us repeat, first of all, that in NRR each transformation admits its inverse transformation[14], as already mentioned. But we have to discuss a more important issue about SLTs, by referring ourselves for simplicity to a superluminal boost along x.

Fig.2 is sketched in Fig.5. It indicates a characteristic of NRR, namely that, if the initial subluminal object has sizes Δx, Δy and Δz (determined by its intersections with

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the space axes), the corresponding superluminal object moving along x with speed V has along x the size $\Delta x' = \Delta x \sqrt{V^2 - 1}$, regularly given by the generalized Lorentz contraction formula[14]. But it does not have real intersections with the transverse Cartesian axes, so that[14] (forgetting here for the double sign) it is endowed with $\Delta y' = i\, \Delta y$; and $\Delta z' = i\, \Delta z$. We see how in the transverse space directions one appears to be formally in need of imaginary units —as implied by our previous Eqs.(3) and (4)— to represent the fact that (passing on to the limiting case) a point is transformed into a double cone. In any case, the SLTs, and the GLTs, provided by NRR, can be found for instance in pages 123–126 of Ref.[14], while the elegant SLTs in 2D can be found, e.g., in pages 26–30 of the same review [14] (or in Ref.[26]).
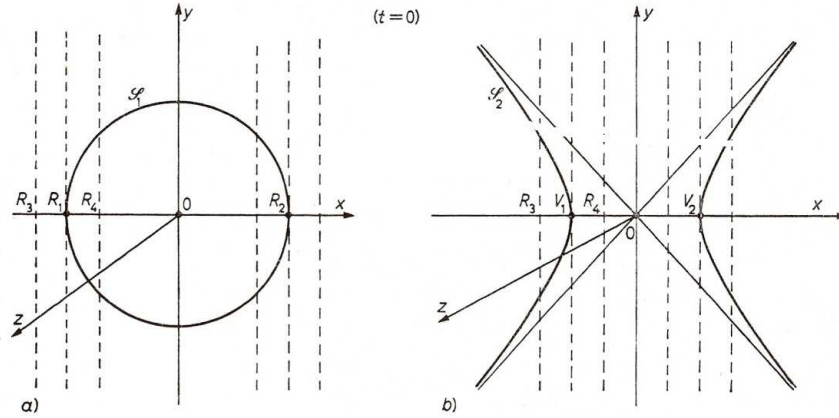

Figure 5: Sketch of Fig.2 showing that, if the initial subluminal object has sizes Δx, Δy and Δz (determined by its intersections with the space axes), the corresponding super-luminal object moving along x with speed V has along x the size $\Delta x' = \Delta x \sqrt{V^2 - 1}$, regularly given by the generalized Lorentz contraction formula[14]. *But* it does not have real intersections with the transverse Cartesian axes, so that[14] (forgetting here for the double sign) it is characterized by $\Delta y' = i\, \Delta y$; and $\Delta z' = i\, \Delta z$. See the text.

A further observation is the following: From Fig.5, and Eq.(3), one realizes the remarkable fact that a SLT will just transform the imaginary part of a subluminal geometric object into the real part of the transformed superluminal object, and vice versa, in the simple

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typical case of the Lorentz transformation "dual" of the identity transformation ($v = 0$) and therefore corresponding to $V = \infty$ as provided by the standard duality relation $V \equiv 1/v$. Remember[14] that in NRR only the speed c of light is invariant (and not the infinite speed!)

In conclusion, according to NNR, an imaginary unit, besides a double sign, must formally enter in the transverse components of Eqs.24 and 25 in Ref.[1]. A mathematically sound way for expressing the SLTs is the only (formal) problem left with NNR, and we are glad of the renewed interest in it with the hope that some scholar will tackle it; one of the authors (ER) and his collaborators have not succeeded yet in finding out a really satisfactory formal expression, even by using Clifford algebras (unless one goes from four to six dimensions). [A similar probem is actually met also when passing on from the exterior to the interior of a black-hole (BH), using not ρ and t only, but four coordinates; and even more in the case of a non-spherically symmetric BH].

Nevertheless, one can usefully apply our SLTs. As realized also by Seshadri[1], in the particular case of the complete Maxwell equations, which are covariant under rotations in the 6D space ($\boldsymbol{E}, \boldsymbol{H}$) and in particular under the transformations $\boldsymbol{E} \to i\,\boldsymbol{H}$; $\quad \boldsymbol{H} \to -i\,\boldsymbol{E}$, one can obtain the generalized Maxwell equations[14,28] by meaningfully using the SLTs in their present form (given for instance in pages 123–126 of Ref.[14]). Our generalized Maxwell equations are presented in Sec.6.15, and particularly in Sec.15 of Ref.[14] and references therein (for example, see Ref.[28]).

Even more, one can have useful recourse to our SLTs, merely by remembering that they *change sign to the quadratic forms* $x_\mu x^\mu$, $p_\mu p^\mu$, and $x_\mu p^\mu$. This has been done in Refs.[18,14] to get, e.g., the result shown in our Fig.2. Indeed, to get the shape of a simple tachyon (the transform of a quite simple, ball-shaped bradyon moving in the

x-direction), one starts with the world-tube associated in spacetime with the considered, spherical, subluminal object. For subluminal speeds v, the world-tube will be inclined with respect to the t-axis of an angle $|\alpha| < 45°$; and, by cutting it with a plane $t = $ constant, one gets of course the ellipsoid produced by the Lorentz space-contraction along x. In the case of a superluminal speed V, the world-tube happens to be outside the light cone,where the geometry is pseudo-Euclidean. Therefore, we wrote down in manifestly covarint form the equation of the cylindrical surface of the subluminal tube, and transformed it superluminally: That is to say, by changing sign to the quadratic forms entering the said equation. Then, by cutting with $t = $ constant the resulting new equation, we easily obtained[18,14] the shape of the corresponding superluminal object, which resulted to be the X-shaped region included between an indefinite double-cone and a two-sheeted hyperboloid (see Fig.2).

2.5 Reply to Seshadri's Section 6

In Sec.6 of Ref.[1], our publication [7] is re-examined. In that paper, we just investigated the toy-model of a point-like superluminal charge (a very bad approximation in all cases, and particularly in the tachyonic case, always leading as we know to divergencies), and we carefully mentioned that in such a case one *has* to meet singularities on the double cone. The corresponding claims by Seshadri just confirm our own statements[7]. Of course, for the reasons previously discussed, calculations suitably performed in order to represent half retarded and half advanced fields yield a double-cone geometry.

3. Acknowledgements

3. Bibliography

[1] S.R.Seshadri, "Cherenkov radiation versus X-shaped localized waves: Comment," J. Opt. Soc. Am. A **29**(12), 2532-2535 (2012): previous article.

[2] M.Zamboni-Rached, E.Recami and I.M.Besieris, "Cherenkov radiation versus X-shaped localized waves," J. Opt. Soc. Am. A **27**, 928-934 (2010).

[3] S.C.Walker and W.A.Kuperman, "Cherenkov-Vavilov formulation of X waves," Phys. Rev. Lett. **99**, 244802 [four pages] (2007).

[4] *Localized Waves, Theory and Applications*, ed. by H.E.Hernández-Figueroa, M.Zamboni-Rached, and E.Recami (J.Wiley; New York, 2008).

[5] J.-y. Lu and J.F.Greenleaf, "Experimental verification of nondiffracting X-waves," IEEE Trans. Ultrason. Ferroelec. Freq. Control **39**, 441-446 (1992).

[6] P.Saari and K.Reivelt, "Evidence of X-shaped propagation invariant localized light waves," Phys. Rev. Lett. **79**, 4135-4138 (1997).

[7] E.Recami, M.Zamboni-Rached and C.A.Dartora, "Localized X-shaped field generated by a superluminal charge," Phys. Rev. E **69**, 027602 [four pages] (2004); and refs. therein.

[8] E.Recami and M.Zamboni-Rached: "Localized Waves: A Review", Advances in Imaging & Electron Physics (AIEP) **156**, 235-355 (2009) [121 printed pages).

[9] A.Sommerfeld, "Überlichtgeschwindigkeitsteilchen," K. Ned. Akad. Wet. **8**, 346 (1904).

[10] A.Sommerfeld, "Zur Electronentheorie (3 Tiele)," Nach. Kgl. Ges. Wiss. Göttingen,

Math. Naturwiss. Klasse 99-130, 363-439 (1904), 201-236 (1905).

[11] P.O. Fröman, "Historical background of the tachyon concept," Arch. Hist. Exact Sci. **48**, 373-380 (1994); DOI: 10.1007/BF00375087.

[12] O.-M.Bilaniuk, V.K.Deshpande and E.C.G.Sudarshan, " 'Meta' relativity," Am. J. Phys. **30**, 718-723 (1962).

[13] O.-M.Bilaniuk and E.C.G.Sudarshan, "Particles beyond the light barrier," Phys. Today **22**, 331-339 (1969).

[14] E.Recami, "Classical theory of tachyons," Rivista Nuovo Cim. **9**(6), 1-178 (1986), monographic issue no.6.

[15] R.Mignani and E.Recami: "Crossing Relations Derived from (Extended) Relativity", Int. J. Theor. Phys. **12**, 299-320 (1975).

[16] M.Pavšič and E.Recami: "Charge Conjugation and Internal Space-Time Symmetries", Lett. Nuovo Cim. **34**, 357-362 (1982).

[17] E.Recami: "Tachyon Mechanics and Causality; A Systematic Thorough Analysis of the Tachyon Causal Paradoxes," Found. Phys. **17**, 239-296 (1987).

[18] A.O.Barut, G.D.Maccarrone and E.Recami: "On the shape of tachyons," Nuovo Cimento A **71**, 509-533 (1982).

[19] R.Mignani and E.Recami: "Tachyons do not emit Cherenkov radiation in vacuum," Lett. Nuovo Cim. **7**, 388-390 (1973).

[20] A.B.Utkin, "Droplet-shaped waves: causal finite-support analogs of X-shaped waves," J. Opt. Soc. Am. A **29**, 457-462 (2012).

[21] P.M.Morse, *Theoretical Acoustics* (Princeton Univ. Press, 1985).

[22] E.Arias, C.H.G.Bessa and N.F.Svaiter, "An analog fluid model for some tachyonic effects in Field Theory," Mod. Phys. Lett. A **26**, 2335-2344 (2011); and refs. tehrein.

[23] E.Recami: "The Tolman antitelephone paradox: Its solution by tachyon mechanics," 1985, reprinted in Electronic J. Theor. Phys. (EJTP) **6**(21), 1-8 (2009).

[24] E.Recami: "Superluminal motions? A bird's-eye view of the experimental status-of-the-art", Found. Phys. **31**, 1119-1135 (2001).

[25] E.Recami: "Superluminal waves and objects: An up-dated overview of the relevant experiments", arXiv:0804.1502 [physics].

[26] E.Recami and W.A.Rodrigues: "A model theory for tachyons in two dimensions," in *Gravitational Radiation and Relativity* (vol.3 of the Proceedings of the Sir Arthur Eddington Centenary Symposium), ed. by J.Weber and T.M.Karade (World Scientific; Singapore, 1985), pp.151-203.

[27] A.O.Barut and Chandola: "Localized' tachyonic wavelet solutions to the wave equation," Phys. Lett. A **180**, 5-8 (1993).

[28] E.Recami and R.Mignani: "Magnetic Monopoles and Tachyons in Special Relativity," Phys. Lett. B **62**, 41-43 (1976).